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The Effect of Uncertainty Principle on the Thermodynamics of Early Universe

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We discuss the concept of measurement in cosmology from the relativistic and quantum mechanical points of view. The uncertainty principle within the particle horizon, excludes the momentum of particles to be less than $\pi\hbar H/c$. This effect modifies the standard thermodynamics of early universe for the ultra-relativistic particles such that the equation of state as well as dependence of energy density and pressure to the temperature. We show that this modification to the thermodynamics of early universe is important for energies $E > 10^{17}GeV$. During the inflation, the particle horizon inflates to a huge size and makes the uncertainty in the momentum to be negligible.

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The concept of measurement is the basis of two main theories of physics, relativity and quantum mechanics. At the present universe where the cosmological scales are too large and energy density of cosmological fluid is too low, the measurement

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can be treated classically. However, for the early universe with higher energies and smaller scales, using quantum mechanical concept in the measurement is essential. As we know from the basis of quantum mechanics, if we confine a particle inside a box, an uncertainty due to this confinement appears in the momentum space. A well known example is the scattering of the particles from the slit, where the confinement of the position causes uncertainty in the momentum, resulting the formation of the scattering pattern. Analogue to a particle inside a box, in the case of the early universe we have a causal box (i.e. particle horizon) which any observer in the universe is confined to do his/her measurements within this scale [1]. In another word considering a wave function to a particle, the probability of finding a particle by an observed outside its horizon is zero, $\Psi^2(\lambda > horizon) = 0$.

From the theory of relativity, measurement of a stick length can be done by sending simultaneous signals to the observer from the two endpoints, where for the scales larger than the causal size, those signals need more than the age of the universe to be received. Looking back to the history of the universe, the particle horizon after the Planck era grows as cH^{-1} but inflates to a huge size by the beginning of inflationary epoch [2]. In the pre-inflationary epoch the maximum uncertainty in the location of a particle cH^{-1} results in a minimum uncertainty in the momentum as:

$$p_{min} = \pi\hbar H/c, \tag{1}$$

where H is the Hubble parameter.

This uncertainty can also be described by an experiment similar to the Heisenberg Gedanken microscope. Using photons with small wavelengths to probe the position of a particle makes larger uncertainty in the momentum. To minimize this effect, we have to use photons as large as possible wavelengths. However photons with the wavelengths larger than the particle horizon are not detectable. The horizon size wavelength photons are the weakest accessible probe, producing a minimum uncertainty of $p = \pi\hbar H/c$. As we go back further to the Planck epoch the size of particle horizon becomes smaller and consequently the uncertainly in momentum raises.

For the ultra-relativistic particles the minimum uncertainty of energy in three spatial dimensions is $E_{min} = \sqrt{3}\pi\hbar H$. To have a feeling from the amount of uncertainty in the energy we compare it with the thermal energy through E_{min}/kT. This ratio grows as we go further to the early universe. In the Planck epoch $kT \simeq M_{pl}$ and $H^2 \simeq M_{pl}{}^2$, so these two energies tend to the same order of magnitude.

Excluding the momenta smaller than $P_{min} = \pi\hbar H/c$ from the phase space, modifies the standard thermodynamics of early universe. We start with the partition function for the fermions and bosons to calculate thermodynamical parameters;

$$\ln Z = \pm\sum_n \ln(1 \pm e^{-\beta E_n}), \tag{2}$$

where $\beta = 1/kT$ and upper and lower signs stand for the fermions and bosons, respectively. For the ultra-relativistic particles with the continuum energy spectrum,

the partition function can be written as:

$$\ln Z = \pm g \int_{E_{min}}^{\infty} \frac{4\pi n^2}{8} \ln(1 \pm e^{-\beta E_n}) dn,$$

$$= \mp \frac{gV}{6\pi^2 \hbar^3 c^3} E_{min}^3 \ln(1 \pm e^{-\beta E_{min}})$$

$$+ \frac{\beta gV}{6\pi^2 \hbar^3 c^3} \int_{E_{min}}^{\infty} \frac{E^3 dE}{e^{\beta E} \pm 1}, \tag{3}$$

where $E_n = n\pi\hbar H$, $4\pi n^2/8$ in the first integrand is the density of states, g is the degree of freedom for a quantum state and V is the volume of the causal box. We use standard thermodynamics, $P = \frac{kT}{V} \ln Z$ to calculate the pressure as follows:

$$P = \frac{\rho c^2}{3} \mp \frac{g}{6\pi^2 \hbar^3 c^3 \beta} E_{min}^3 \ln(1 \pm e^{-\beta E_{min}}), \tag{4}$$

where the first term at the right hand side of equation of state comes from the energy density as:

$$\rho = \frac{g}{2\pi^2 \hbar^3 c^5} \int_{E_{min}}^{\infty} \frac{E^3 dE}{e^{\beta E} \pm 1}, \tag{5}$$

and the second term is the correction term to the pressure, say it P_c. Fig. (1) shows the deviation of Eq. (4) from the standard definition of pressure in terms of E_{min}/KT. Close to the Planck epoch $E_{min}/KT \to 1$ and the deviation from the standard definition of pressure, P_c/P is $\sim 5.4\%$ for the fermions and $\sim 7\%$ for the bosons.

The dependence of the mass density of fluid of fermions and bosons to the temperature also can be obtained according to the Eq. (5) as:

$$\rho_f = \frac{7g_f \pi^2 K^4 T^4}{240 c^5 \hbar^3} - \frac{g_f}{240 c^5 \hbar^3 \pi^2} [30 E_{min}^4 - 7\pi^4 K^4 T^4$$

$$- 120 E_{min}^3 KT \log(1 + e^{\frac{E_{min}}{KT}}) + 360 E_{min}^2 K^2 T^2$$

$$\times \sum_{n=1}^{\infty} \frac{e^{\frac{nE_{min}}{KT}}}{n^2} - 720 E_{min} K^3 T^3 \sum_{n=1}^{\infty} \frac{e^{\frac{nE_{min}}{KT}}}{n^3}$$

$$+ 720 K^4 T^4 \sum_{n=1}^{\infty} \frac{e^{\frac{nE_{min}}{KT}}}{n^4}]. \tag{6}$$

$$\rho_b = \frac{g_b \pi^2 K^4 T^4}{30 c^5 \hbar^3} - \frac{g_b}{120 c^5 \hbar^3 \pi^2} [-15 E_{min}^4 - 4\pi^4 K^4 T^4$$

$$+ 60 E_{min}^3 KT \log(1 - e^{\frac{E_{min}}{KT}}) + 180 E_{min}^2 K^2 T^2$$

$$\times \sum_{n=1}^{\infty} \frac{e^{\frac{nE_{min}}{KT}}}{n^2} - 360 E_{min} K^3 T^3 \sum_{n=1}^{\infty} \frac{e^{\frac{nE_{min}}{KT}}}{n^3}$$

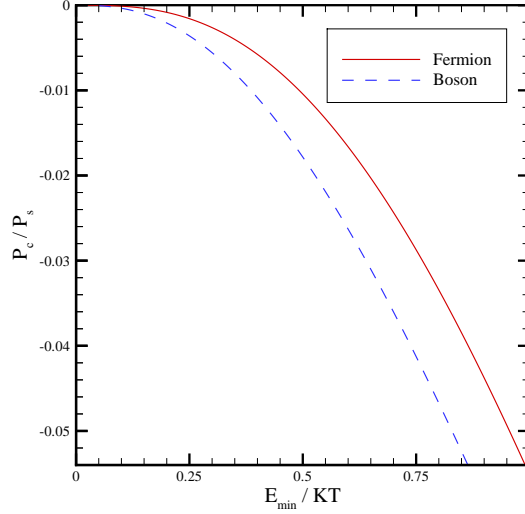$$+ 360 K^4 T^4 \sum_{n=1}^{\infty} \frac{e^{\frac{nE_{min}}{KT}}}{n^4}]. \tag{7}$$

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Fig. 1.   The ratio of the correction term of the pressure to the of standard definition of the pressure as a function of E_{min}/kT. For the low energies, this ratio tends to zero.

If we let $E_{min} = 0$, we will get the familiar relation between the mass density and temperature for the ultra-relativistic fermions and bosons [3]. The density deviation from the standard thermodynamics $\frac{\rho_s - \rho}{\rho_s}$ is shown in Fig. (2). For temperatures near the Planck epoch, the density is lower than that of standard one. For the entropy density calculation $s = -\frac{1}{V} \times \partial F/\partial T$ we use the free energy $F = -kT \ln Z$ which results in:

$$s = \frac{4\rho c^2}{3T} \mp \frac{g}{6\pi^2 \hbar^3 c^3 \beta} E_{min}^3 \ln(1 \pm e^{-\beta E_{min}}). \tag{8}$$

The second term is the correction to the standard thermodynamics and grows as we approach to the early universe.

The equation of state can be obtained by substituting Eqs. (4) and (5) in the continuity equation $\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P/c^2) = 0$, as follows:

$$\begin{aligned}
\frac{d\rho}{da} &= -\frac{3}{a}(\rho + P/c^2), \\
&= -\frac{3}{a}(\frac{4}{3}\rho + P_c/c^2).
\end{aligned} \tag{9}$$

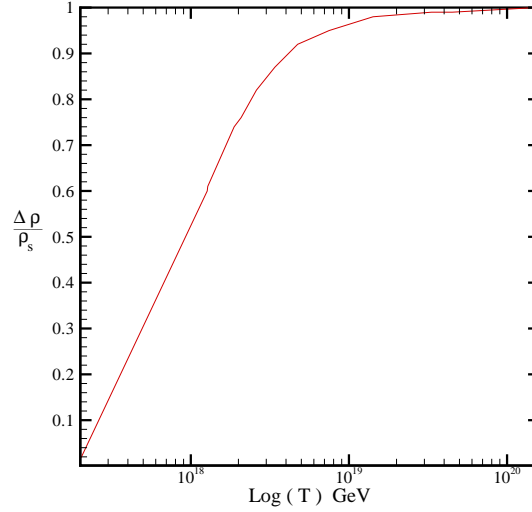We use the iteration method and apply the standard thermodynamics to solve the

Fig. 2.   Relative difference between the standard energy density ρ_s and that of considering the uncertainty correction ρ, $(\frac{\rho_s - \rho}{\rho_s})$ as a function of $\log(T)$ in GeV.

continuity equation up to the first order correction for the fermions and bosons as:

$$\frac{d\varrho_f}{da} = -\frac{4}{a}\left[\varrho_f - 46.8g^{3/4}a^{-6}\varrho_f^{1/4}\right],\tag{10}$$

$$\frac{d\varrho_b}{da} = -\frac{4}{a}\left[\varrho_b - 389.6ga^{-8}\right],\tag{11}$$

where ϱ is normalized to the Planck energy density. The analytical solutions of Eqs. (10) and (11) are:

$$\varrho_f = \left[a^{-3} + 46.8g^{3/4}(a^{-3} - a^{-6})\right]^{3/4},\tag{12}$$

$$\varrho_b = a^{-4} + 389.6g(a^{-4} - a^{-8}).\tag{13}$$

Here we have deviation from the standard dependence of density to the scale factor as $\rho \propto a^{-4}$. To obtain the dependence of the scale factor to the temperature we use the solutions of the Eqs. (12) and (13) at the left hand side of Eqs. (6) and (7) and ignore the second order corrections. The deviation of temperature from $1/a$ for the fermions and bosons is shown in Fig. (3), where it behaves as $1/a^n$ with the index of $n \simeq 1.5$.

In summery we have shown that a natural constraint to the lower limit of particles energy appears due to their confinement at the particle horizon. The effect is the modification of the thermodynamics of the very early universe for energies $E > 10^{17}GeV$. In consequence, the equation of state and dependence of temperature, pressure, energy density to the scale factor are all modified. Inflating the size of the
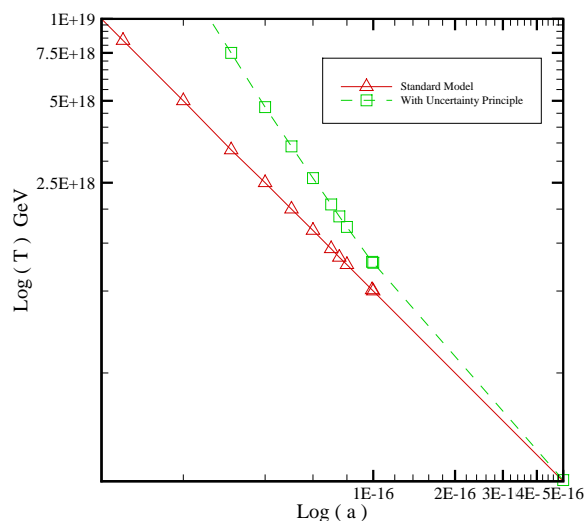
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Fig. 3.   The temperature as a function of the scale factor, in logarithmic scales. The solid line indicates this dependence in the normal thermodynamics of universe ($T \propto 1/a$) and the dashed line is for the case that we use the uncertainty principle. Here the scale factor $a = 1$ is chosen for the electroweak energy.

particle horizon during the inflation, makes the uncertainty of the momentum to be negligible. This effect may shed light on the thermodynamics of early universe considering the cosmic fluid generated from the energy momentum of quantum fields [4].

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